Precalculus, Quarter 2, Unit 2.1

Binomial Theorem and Polynomial Expansion

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Expand binomials using Pascal's triangle.
- Develop binomials using binomial theorem.
- Generate Pascal's triangle.
- Calculate a given term utilizing Pascal's triangle.
- Compute a given coefficient using binomial theorem.
- Produce the original binomial given a term or an expanded binomial with leading coefficient of 1.

Mathematical practices to be integrated

Reason abstractly and quantitatively

- Connect the expanded form of the binomial to Pascal's triangle.

Attend to precision

- Construct the coefficients with the binomial theorem calculations and correct Pascal rows.

Look for and make use of structure

- Identify the degree, term, row, coefficient from the pattern of Pascal.

Essential questions

- How does Pascal's triangle help us to expand binomials?
- What does binomial theorem do to help us in binomial expansion?
- What are similarities and differences between using Pascal's triangle versus Binomial theorem?

Written Curriculum

Common Core State Standards for Mathematical Content

Arithmetic with Polynomials and Rational Expressions	A-APR

Use polynomial identities to solve problems.

A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

[1] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 3 students multiplied within 100. In grade 4 students used multiplication to compare and fractions by whole numbers. In grade 5, students extended multiplication to include fractions with themselves. In grade 6 students multiply multi-digit decimals. In grade 7 students applied the distributive property. In algebra 1 students factored quadratics, and worked with laws of exponents. In algebra 2 students have graphed polynomial functions and students have worked with series and sigma notation for geometric series.

Current Learning

Students will be expanding factors of polynomial functions using Pascal's triangle and binomial theorem.

Future Learning

Students will be using binomial expansion and modified Pascal triangles in Number theory and Calculus, Statistics with binomial distributions.

Additional Findings

Students struggle with creating the triangle, with understanding the relationship between the triangle entries and the coefficients of the polynomial functions. Students have trouble with evaluating sigma notation.

Precalculus, Quarter 2, Unit 2.2

Performing Operations with Complex Numbers

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Perform arithmetic operations on complex numbers.
- Graph complex numbers on the complex plain.
- Graph polar coordinates on the polar coordinate system.
- Use complex conjugates to simplify complex numbers.
- Factor polynomial functions to find the complex roots.
- Calculate the distance and midpoint on the complex plane.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others

- Relate how to perform distance and midpoint calculations on the new complex number system from the Cartesian coordinate system.

Look for and make use of structure

- Connect the rectangular and polar forms of the complex number system by distance and angular polar form.

Look for and express regularity in repeated reasoning

- Perform factoring to find complex roots from previous related material.

Essential questions

- How are the rectangular and polar forms of complex numbers related?
- Why is the complex conjugate important in calculations of complex numbers?
- How are distance and midpoint calculations of complex numbers similar and different from distance and midpoint of Cartesian coordinates?
- What is magnitude and how does it relate to graphing in complex number systems.

Written Curriculum

Common Core State Standards for Mathematical Content

The Complex Number System	**N-CN**

Perform arithmetic operations with complex numbers.

N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument* 120°.

N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

N-CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In algebra 1, students learned about imaginary numbers. In geometry students calculated distances and midpoints. In algebra 2, students worked with complex roots of quadratics and polynomials.

Current Learning

Students simplify fractions with complex conjugates. Students will factor a polynomial function completely to find complex roots. Students will be graphing complex numbers on the complex plane and on the polar coordinate system. Students will perform operations of addition, subtraction, multiplication, and division on complex numbers. Students will perform distance and midpoint calculations with the complex number system.

Future Learning

Students will use complex numbers in circuitry and electrical engineering. Students will use the complex number system to explain elementary particles and their behaviors in theoretical and particle physics.

Additional Findings

Students struggle with the concept of the real versus imaginary portion of the number. Students struggle with the concept of imaginary numbers. Students have trouble with many fundamental square root simplifications and operations.

Precalculus, Quarter 2, Unit 2.3

Building Sequences and Series

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Write and use arithmetic and geometric sequences explicitly.
- Write and use arithmetic and geometric sequences recursively.
- Use arithmetic and geometric sequences *and series** to model situations.
- Translate between arithmetic and geometric forms.
- *Calculate the sum of an infinite series.*
- *Distinguish between converging and diverging infinite geometric series.*
- *Convert between series and sigma notations.*

Mathematical practices to be integrated

Model with mathematics.

- Analyze a population growth model and apply appropriate formulas.
- Utilize appropriate formulas on depreciation models.
- Interpret linear models in terms of arithmetic sequences.

Make sense of problems and persevere in solving them.

- Determine the type of sequence *and series* and apply the correct formula.

Use appropriate tools strategically.

- Use STATPLOT, ZoomStat Summation ∑(, LinReg(ax+b), and ExpReg.

Look for and make use of structure.

- Recognize patterns in various arithmetic and geometric relationships, using number sense.

Essential questions

- *What are the differences between arithmetic and geometric sequences and series?*
- What are the similarities and differences between recursive and explicit formulas?
- *When does a series converge or diverge?*
- *When and why is it appropriate to use sigma notation?*

*Italics are used to indicate that the italicized content statement, practice statement, or essential question does not appear in the Common Core State Standards for Mathematics.

Written Curriculum

Common Core State Standards for Mathematical Content

Building Functions	**F-BF**

Build a function that models a relationship between two quantities [*Include all types of functions studied*]

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten through third grade, students were introduced to counting sequences. In fourth and fifth grade, students analyzed patterns. In Algebra I, students learned to recognize and use formulas of arithmetic and geometric sequences. In Algebra II, students learned how arithmetic and geometric sequences are related to linear and exponential functions.

Current Learning

Students learn to write and translate between arithmetic and geometric explicit and recursive formulas. Students model real-world problems using arithmetic and geometric forms. With respect to series, students convert between sigma and series notation. Students learn to recognize and apply formulas to infinite geometric series.

Future Learning

In Calculus, students will have to work with rates of change in terms of slope of tangent lines and infinite series. In Statistics, students will perform linear regressions and apply those models to data sets.

Additional Findings

Sequences can cause problems for students because they are not able write the sequences in explicit or recursive form. Teachers have difficulties teaching how to write a sequence in explicit and recursive forms, as well as when to apply each form appropriately in a real-life context. When working with series, students have difficulty converting between sigma and series notation. Students also have difficulties calculating a sum when given sigma notation.